

EC 131 - Algebraic solution of elasticities

Boston College
Department of Economics

Inacio Guerberoff Lanari Bo

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1 Price elasticity of demand

Suppose that the demand function is the following, for $a > 0$ and $b \geq 0$:

$$Q = a - bP$$

We know that the demand elasticity is given by the following:

$$\varepsilon_{P,Q} = \frac{\Delta Q}{\Delta P} \frac{P}{Q}$$

In a linear demand function, the value of $\frac{\Delta Q}{\Delta P}$ is the slope of the demand function $Q(P)$, which here is $-b$. As usual, we will ignore the negative sign of this coefficient for demand elasticities. Thus:

$$\varepsilon_{P,Q} = b \frac{P}{Q}$$

If we want to know the value of the elasticity as a function of the price, we should just replace Q by $a - bP$:

$$\varepsilon_{P,Q} = \frac{bP}{a - bP}$$

Note: when $P = 0$, $\varepsilon_{P,Q} = 0$ and when $P = a/b$, $\varepsilon_{P,Q} \rightarrow \infty$.

If we want to know the value of the elasticity as a function of the price, we should now replace P by the inverse demand function $P(Q)$, which we can find directly from the demand function:

$$bP = a - Q$$

$$P = \frac{a - Q}{b}$$

That is:

$$\varepsilon_{P,Q} = b \frac{P}{Q} = b \frac{\frac{a-Q}{b}}{Q} = \frac{a-Q}{Q}$$

Note: when $Q = a$, $\varepsilon_{P,Q} = 0$, and when $Q = 0$, $\varepsilon_{P,Q} \rightarrow \infty$.