

# EC 131 - Algebraic solution of elasticities

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## 1 Price elasticity of demand

Suppose that the demand function is the following, for  $a > 0$  and  $b \geq 0$ :

$$Q = a - bP$$

We know that the demand elasticity is given by the following:

$$\varepsilon_{P,Q} = \frac{\Delta Q}{\Delta P} \frac{P}{Q}$$

In a linear demand function, the value of  $\frac{\Delta Q}{\Delta P}$  is the slope of the demand function  $Q(P)$ , which here is  $-b$ . As usual, we will ignore the negative sign of this coefficient for demand elasticities. Thus:

$$\varepsilon_{P,Q} = b \frac{P}{Q}$$

If we want to know the value of the elasticity as a function of the price, we should just replace  $Q$  by  $a - bP$ :

$$\varepsilon_{P,Q} = \frac{bP}{a - bP}$$

Note: when  $P = 0$ ,  $\varepsilon_{P,Q} = 0$  and when  $P = a/b$ ,  $\varepsilon_{P,Q} \rightarrow \infty$ .

If we want to know the value of the elasticity as a function of the price, we should now replace  $P$  by the inverse demand function  $P(Q)$ , which we can find directly from the demand function:

$$bP = a - Q$$

$$P = \frac{a - Q}{b}$$

That is:

$$\varepsilon_{P,Q} = b \frac{P}{Q} = b \frac{\frac{a-Q}{b}}{Q} = \frac{a-Q}{Q}$$

Note: when  $Q = a$ ,  $\varepsilon_{P,Q} = 0$ , and when  $Q = 0$ ,  $\varepsilon_{P,Q} \rightarrow \infty$ .